

Radford, L. (2024). Self and Other. In P. Ernest (Ed.), *Ethics and mathematics education: The good, the bad and the ugly* (pp. 35-53). Springer.

Self and Other

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In premodern times, people didn't speak of "identity" ... not because people didn't have (what we call) identities ... but rather because [identity was] then too unproblematic to be thematized as such. (Taylor; 2003, p. 48)

Abstract In this chapter, I discuss two different contemporary perspectives that attempt to reimagine self and other beyond the conundrums of modernity. These perspectives are interesting to examine as they try to move us beyond the societal predicament in which we are immersed today, and as conceptions of self and other constitute the background of ethical theories. The first one comes from Charles Taylor as articulated in his ethics of authenticity. The second one comes from dialectical materialism—in particular, from Pierre Macherey's and Franck Fischbach's studies on Marx—and Étienne Babilar's work on politics, culture, and identity. In the last part of the chapter, I discuss some implications to the reconceptualization of self and other in the mathematics classroom.

Keywords Ethics · Authenticity · Dialectical materialism · Politics · Culture · Identity

Introduction

Around 1275, the Dominican friar Jacobus de Cessolis finished a political treatise out of successful sermons he delivered in the Lombard region of Italy: The *Liber de moribus hominum et officiis nobilium* (Book of the Manners of Men and the Offices

of the Nobility). One of the distinct characteristics of this treatise is that it uses the chessboard and its pieces as a metaphor to depict a political organization of society.¹

The true peculiarity of Jacobus's treatise lies, however, elsewhere. While other treatises have portrayed late medieval societies through organicist metaphors, Jacobus does not. In the organicist metaphor, society is considered like a body, where each part has its own position and role to play. For example, in John of Salisbury's *Policraticus*, written in 1159, the state appears as "a sort of body which is animated by the grant of divine reward and which is driven by the command of the highest equity and ruled by a sort of rational management" (cited in Adams, 2009, p. 3). By contrast, Jacobus's treatise draws on the game of chess to imagine society rather as a place where individuals are tied to each other by *obligations*. These obligations are of a *contractual* nature. Jacobus presents "the idea of a kingdom organized around professional ties and associations, ties that are in turn regulated by moral law, rather than around kinship" (Adams, 2009, p. 1). So, in Jacobus's *Liber*:

each piece corresponds to a specific professional identity, with all pieces being interdependent; just as the king needs the blacksmith, represented by the pawn before his square, so farmers depend on the protection of the knights, who are found on an adjacent square. (Adams, 2009, p. 3)

There are a few points that deserve our attention in the medieval conceptions of society that may shed some light on the central question of this chapter; that is, the question of self and other. Let us start by noticing that the question of self or the question of the self's identity is not addressed in medieval treatises. The *nature* of identity or the *nature* of the self was not a problem for medieval subjects. As Taylor (2003) notes, this was not because individuals in the Middle Ages did not have identities or were not occupied with their own being. The question simply did not arise. The question of the self is a question of modernity. For such a tremendous question to arise, new societal processes and the concomitant new forms of social consciousness needed to be put in place. Our sense of self—how each one of us comes to experience our self in our daily life, the manners in which we come to see ourselves and others—are framed and distributed across the various processes of society: cultural, political, economic, aesthetic, and so on. Jacobus's *Liber*, with the new vision of the state as regulated by rules rather than by the natural transcendental order of the organicists, hints, indeed, at the collapse of an old social order and the beginning of a new one. This new order was driven by the spreading of commercial activities that, in the midst of a new "redistribution of political and social capital" (Adams, 2009, p. 5), brought with it, in a lengthy process of secularization, the construction of a new society (Lenoir, 2022). It also brought new forms of experience for the self and the possibilities for what Morris calls "the discovery of the individual" (Morris, 1972).

¹ It does not come as a surprise, hence, that the *Liber* was later on referred to as *The Book of Chess*. This is its title in the modern English translation (see de Cessolis, 2008) as it was the fifteenth century translation made by William Caxton (for a modern edition, see Caxton, 2009).

If modernity could be defined as the Western historical period where the discovery of the individual was first made possible, postmodernity is perhaps the witness to its *échouement*, its crash. The question of otherness seems, indeed, to point to a crisis of identity: the crisis of a transcultural monolithic self that is being questioned, among others, by decolonizing voices that fail to recognize themselves in it (Chronaki & Planas, 2018; León & Zemelman, 1997; Parra, 2018; Sumida Huaman & Martin, 2020; Valencia, 2018). It seems that we have lost the certainty we inherited from Descartes and that we can no longer be defined as *thinking substances* capable of grasping our own essence through the sightings of our own interiority. Descartes's *cogito* has been shattered and we find ourselves, much like Jacobus de Cessolis, at the doors of a new historical period. We now find ourselves in the pursuit of our own meaning. And because we failed to find it *in us*—not because we have not tried hard, not because we have not dug enough—we try, it seems, to find it elsewhere: in the other.

But what happened? What happened is that we embarked on an entirely new social, cultural, and political path where economic life came to affect our ways of being and living, and the aesthetic of the world. The economic sphere re-shaped our communication systems, legal apparatuses, and the circulation of goods. The economic sphere became the *central kernel* around which the other societal processes were organized, affecting in a decisive way how we came to see ourselves.

There was, then, what Gilles Lipovetsky calls *the process of personalization*; that is, the process that “has massively promoted and embodied a fundamental value, that of personal fulfillment, that of respect for subjective singularity, for the incomparable personality, regardless of the new forms of control and homogenization that are simultaneously achieved” (Lipovetsky, 1989). The process of personalization is at the heart of

a new way for society to organize and orient itself, a new way of managing behavior, no longer through the tyranny of details but with the least constraint and the most private choices possible, with the least austerity and the most desire possible, with the least coercion and the most understanding possible. A process of personalization, in fact, in that the institutions are now indexed on motivations and desires. (Lipovetsky, 1989)

With all this, we entered the age of emptiness, the age of the cult of individualism (Lipovetsky, 1989) and ended up losing touch with society and the other. Society has become an ensemble of monads loosely related to each other through simple convenience. This is the view that, in the educational field, has been epitomized with singular force by constructivism and its self-centered view of the individual in general and the student in particular. In constructivism, the other, von Glasersfeld tells us, is a conceptual construction of the self, like a circle, a table, or any object: you are my pure subjective construction. Yet, I need you. I need you because you can do something I cannot: you can corroborate for me the viability of *my* knowledge. As von Glasersfeld put it, “Others have to be considered because they are irreplaceable in the construction of a more solid experiential reality” (1995, p. 127). This way of understanding intersubjectivity cannot end but in seeing others and society in purely instrumental terms—as tools of the self.

So, in the urgent effort to come to grips with our own nature, we now turn to the other in a desperate move that bears the traces of the dislocation of our being. In the otherness of the other, we hope to find ourselves.

However, we have to be cautious. The riddle of the self (Mikhailov, 1980) has failed to find its answer in the theological classical response in terms of soul, substance, and essence. So, the answer would hardly be found in replacing a theology of the essence with another theology of the essence, this time transferred to the other. A prolegomenon to the riddle of the self might consist in overcoming the naturalist secularized contractual conceptions of self and other that started emerging in Jacobus de Cessolis's *Liber*, conceptions that were expanded later by seventeenth century empiricist outlooks of the world and that have come down to us in various guises in pedagogical practice.

In the next sections, I discuss two different contemporary perspectives that attempt to reimagine self and other beyond the conundrums of modernity. These perspectives are interesting to examine as they try to move us beyond the societal predicament in which we are immersed today, and as conceptions of self and other constitute the background of ethical theories (Radford, 2021a, 2023). The first one comes from Charles Taylor as articulated in his *ethics of authenticity*. The second one comes from dialectical materialism and, in particular, from Pierre Macherey's (2008) and Franck Fischbach's (2015) studies on Marx and Étienne Balibar's (1992, 2002, 2014) work on politics, culture, and identity. In the last part of the chapter, I discuss some implications to the reconceptualization of self and other in the mathematics classroom.

The Malaises of Modernity

Taylor identifies what he terms the three malaises of modernity: those “features of our contemporary culture and society that people experience as a loss or a decline, even as our civilization ‘develops’” (2003, p. 1). The first one is about a loss of meaning that results in the fading of moral horizons. “The second concerns the eclipse of ends, in face of rampant instrumental reason. And the third is about a loss of freedom” (Taylor, 2003, p. 10). The common denominator of these malaises is *individualism*.

Taylor acknowledges that individualism is often seen as the finest achievement of modern civilization:

We live in a world where people have a right to choose for themselves their own pattern of life, to decide in conscience what convictions to espouse, to determine the shape of their lives in a whole host of ways that their ancestors couldn't control. And these rights are generally defended by our legal systems. In principle, people are no longer sacrificed to the demands of supposedly sacred orders that transcend them. (Taylor, 2003, p. 2)

Historically speaking, individualism emerged as a form of emancipation: the emancipation of individuals from the structures of feudal times. It was experienced in the nascent late medieval bourgs that hosted guilds of merchants and craftsmen among others, and where labor could be bought and sold. But Taylor notes, such emancipation did not go without its own problems:

Modern freedom was won by our breaking loose from older moral horizons. People used to see themselves as part of a larger order [. . . which] gave meaning to the world and to the activities of social life. The things that surround us were not just potential raw materials or instruments for our projects, but they had the significance given [to] them by their place in the chain of being. (Taylor, 2003, p. 3)

With the appearance of individualism and the ensuing concept of modern freedom came a loss of purpose and the ensuing disenchantment with the world—the first malaise. “People lost the broader vision because they focused on their individual lives” (Taylor, 2003, p. 4). The disenchantment with the world was also precipitated by a new type of rationality that was crucial in the building of modernity: the primacy of instrumental reason—the second malaise of modernity. “By ‘instrumental reason,’” Taylor says, “I mean the kind of rationality we draw on when we calculate the most economical application of means to a given end. Maximum efficiency, the best cost output ratio, is its measure of success” (p. 5). Instrumental reason offers an outlook of the world where things are seen as merchandise and course of actions in terms of cost-benefit analysis. It has led to what Ferreira de Oliveira calls “the ideology of the market;” that is, the “transformation of things, inanimate or alive, in passive elements of commercialization” (Ferreira de Oliveira, in Freire, 2016, p. 113). Within this context,

Nature, water, the air, the earth, the world, the planet, the universe, the human beings, and all other beings, their minds, their organs, their feelings, their sexuality, their beauty, their workforce, their knowledge, their existence, their homes and their lives, are considered as merchandise. (Ferreira de Oliveira, in Freire, 2016, p. 113)

In Taylor’s account, individualism and the imperialism of instrumental reason are connected to another important aspect of contemporary societies: our loss of authentic freedom—the third malaise of modernity. By a peculiar reversal of things, emancipatory freedom turned into its opposite. It turned into its own loss. The loss of authentic freedom consists in our withdrawal from political action and our engagement in genuine social life. Taylor is not alone in pointing to this feature of our time. Rothenberg complains that much as in Ancient Greece contemporary “democracy depoliticizes the political itself” (2010, p. 46). In a similar line of thinking, Michel Freitag notes that nowadays there is no longer a social movement that produces an intelligibility of the social order and politics; there is no longer.

a new general intelligibility of society and history, collective existence, its purposes and its constraints. In a word, there is no longer any politics. Politics, as a reflexivity of living together (this time around the planet, and this time as knowing how to live with techniques), and politics as collective responsibility for the ends of social life, needs to be reinvented. (Freitag, 2002, p. 244)

Drawing on the work of political philosopher Alexis de Tocqueville, Taylor notes that the apathy of politics in our social life

opens the danger of a new, specifically modern form of despotism . . . It will not be a tyranny of terror and oppression as in the old days. The government will be mild and paternalistic. It may even keep democratic forms, with periodic elections. But in fact, everything will be run by an “immense tutelary power,” over which people will have little control. (Taylor, 2003, p. 9)

The apathy of the politics means the “alienation from the public sphere” (Taylor, 2003, p. 10). It is most ironic to see, hence, that the emancipatory project that marked the beginning of modernity and the birth of the freedom of the individual ended up in producing a new form of alienation—the alienation from a communal life—and may explain, if only partially, our attempts for salvation in the search for the other.

Certainly, a society structured around individualism and instrumental reason provides the individuals with freedom. But it is only the freedom of insignificant choices—the freedom to choose the color of our car and everything that subtracts us from the realm of social action. Through the induced political apathy, such a society distracts attention from those decisions that would matter most: the communal participative decision through which we can define with others what a good life and a good society should be.

The Ethics of Authenticity

Is there a way out? Is it possible to defend ourselves against this state of affairs and gain control over the kind of life we want to live? What is the role of education in general, and mathematics education in particular, in such an emancipatory project? The political importance of these questions lies in the fact that “What is threatened here is our dignity as citizens” (Taylor, 2003, p. 10) and our loss of freedom. Taylor reminds us that the only defense against the individualist structures of contemporary societies “is a vigorous political culture in which participation is valued, at several levels of government and in voluntary associations as well” (p. 9). However, the path to follow is mined with difficulties, for

the atomism of the self-absorbed individual militates against this. Once participation declines, once the lateral associations that were its vehicles wither away, the individual citizen is left alone in the face of the vast bureaucratic state and feels, correctly, powerless. This demotivates the citizen even further, and the vicious cycle of soft despotism is joined. (Taylor, 2003, pp. 9–10)

To move beyond the problems of modernity, Taylor articulates a proposal that he terms an *ethics of authenticity*. What does authenticity mean? Taylor starts by acknowledging the historical romantic sense of authenticity; that is, as being true to oneself. This sense of authenticity assumes a concept of the individual according to

which there is something original in each one of us. To be authentic is to attend to this originality, to the inward generation of our own identity. So far, this concept of the individual seems to coincide with the one of constructivism mentioned above: a self-maker individual. We see this concept of the individual in countless official documents of educational policy where the student is portrayed as coming to the world with her own potentialities, the role of the school being then to make sure that the inward generation of the student takes place, or, as our Ministry of Education puts it, to help the student achieve their self-fulfillment and to inspire them “to reach their full potential” (Ontario Ministry of Education [OME], 2014, p. 3). For Taylor, however, authenticity is a *moral idea*. Authenticity acknowledges that “some forms of life are indeed *higher* than others” (Taylor, 2003, p. 17; italics in the original). In doing so, Taylor moves away from the individualist accounts of self, other, and society, where one of the corollaries is that the good life is what each individual seeks. For Taylor, this position leads to an unproductive moral relativism and the atomization of society. He notes that there is something contradictory and self-defeating in the contemporary individualist outlook of society and its ensuing culture of tolerance. This outlook starts with the moral claim that ethics is a subjective stance (an individual’s construction, as von Glasersfeld would say), hence *different* from one individual to another, yet all ethical stances are *equally* good. In the culture of tolerance things “are ultimately just adopted by each of us because we find ourselves drawn to them” (Taylor, 2003, p. 18). Taylor’s point, of course, is not against tolerance, but about *how* tolerance *in this view* comes to be anything but a manifestation of an unaffiliated being that is the product of individualist premises. What worries him about this is that ethics is pushed to the borders of society and vanishes in unarticulated debate. There is no way of articulating differences in meaningful discourses. As Taylor notes, “mere difference can’t itself be the ground of equal value” (p. 51). He reminds us that, unfortunately, “The general force of subjectivism in our philosophical world and the power of neutral liberalism intensify the sense that these [ethical] issues can’t and shouldn’t be talked about” (p. 21).

Dialogical Reason

Now, how are we going to distinguish, among the range of societal forms of life, those that we can call *higher*? For Taylor, there is no answer a priori. The answer is never given; it is not even reachable. The answer is rather the object of a constant and unfinished quest driven by what Taylor calls *dialogical reason*. It is here where we find the Other in Taylor’s account. “Reasoning in moral matters,” he argues, “is always reasoning with somebody” (2003, p. 31). And reasoning in moral matters is part of what should count in the quest of a good life.

It is hence through reasoning that we can enter into dialog with others, that we can voice our perspective, have something to say, and articulate our views.

Articulacy here has a moral point, not just in correcting what may be wrong views but also in making the force of an ideal that people are already living by more palpable, more vivid for them; and by making it more vivid, empowering them to live up to it in a fuller and more integral fashion. (Taylor, 2003, p. 22)

Through the practice of dialogical reason, you can argue about what matters in life, “hence to show that there is indeed a practical point in trying to understand better what authenticity consists in” (Taylor, 2003, p. 32).

Taylor’s proposal comes hence down to three points: “(1) that authenticity is a valid ideal; (2) that you can argue in reason about ideals and about the conformity of practices to these ideals; and (3) that these arguments can make a difference” (Taylor, 2003, p. 23).

These three points are underpinned by what Taylor takes to be the general feature of human life; that is, “its fundamentally dialogical character” (Taylor, 2003, p. 33). Here Taylor comes close to Marx’s idea about human nature and to the dialogical human feature that some Russian scholars, such as Voloshinov (1973) and Bakhtin (1981), have articulated, although with some nuanced differences.

Horizons of Significance

Taylor acknowledges that the three points underlying his ethics of authenticity may be the object of controversy. He notes that people espousing the culture of individualism may go as far as seeing self-other relationships (which are invoked in the dialogical nature of humans) as fulfilling oneself, but not as *defining oneself*. These individuals would say “We will need relationships to fulfill but not to define ourselves” (2003, p. 34). Taylor argues, however, that such a position

forgets how our understanding of the good things in life can be transformed by our enjoying them in common with people we love, how some goods become accessible to us only through such common enjoyment . . . If some of the things I value most are accessible to me only in relation to the person I love, then she becomes internal to my identity. (Taylor, 2003, p. 34)

The argument, however, is in conflict with the romantic view of the authentic self according to which “Defining myself means finding what is significant in my difference from others” (Taylor, 2003, pp. 35–36). To move beyond this tension, Taylor argues that in defining oneself the question is not about the number of hairs on our head or any other empirical trait. It is about our ability to articulate important truths. In short, “identity,” defining oneself, is a matter of *significance*. And significance moves against a background of intelligibility—what Taylor calls a *horizon*. The horizon holds values. Ignoring this fact leads us straightforward to the self-defeating relativism mentioned before—the relativism of the culture of individualism. In this culture, Taylor notes.

All options are equally worthy, because they are freely chosen, and it is choice that confers worth ... But this implicitly denies the existence of a pre existing horizon of significance, whereby some things are worthwhile and others less so, and still others not at all, quite anterior to choice. (2003, pp. 37–38)

As we saw before, Taylor places the link between self and other in their dialogical reasonings; here he goes one step further and places self and other at the crossroad of the free choices they make *and* a horizon of significance that transcends *both*. The horizon of significance endows self's and other's individual choices with meaning.

Even the sense that the significance of my life comes from its being chosen—the case where authenticity is actually grounded on self determining freedom—depends on the understanding that *independent of my will* there is something noble, courageous, and hence significant in giving shape to my own life. (Taylor, 2003, p. 39; italics in the original)

You, I, our neighbor, we all live in a horizon of important questions, a horizon without which we cannot define ourselves significantly, and this horizon is *not* of our making. It is already given. We may come to modify it through our deeds, through our dialogical postures, but this horizon is not entirely of our choice.

Recognition

Taylor is aware that this way of putting things leads to a weak conception of the social: “it encourages a purely personal understanding of self-fulfilment, thus making the various associations and communities in which the person enters purely instrumental in their significance [and] is antithetical to any strong commitment to a community” (Taylor, 2003, p. 43). By definition, authenticity, along with its claim that it must be inwardly generated, “distances us from our relations to others” (p. 44). Here, we are set back to square one, to the constructivist view of self and other (see von Glasersfeld, 1995). Yet, Taylor hopes to overcome the shortcoming alluding to authenticity as a moral principle. In the footsteps of Hegel (1977), the moral principle is materialized in the *recognition of* and *by* others, an idea that underpins the eighteenth century idea of equity and its contemporary version of dignity: the dignity of all human beings.

The crux of the argument is to understand that the inward generation of our own identity is interwoven with a negotiated dialog with others. In the theoretical understanding of authenticity, the identity of the self is to be *discovered*: “My discovering my identity doesn’t mean that I work it out in isolation but that I negotiate it through dialogue, partly overt, partly internalized, with others . . . My own identity crucially depends on my dialogical relations with others” (Taylor, 2003, pp. 47–48). In this context, “Artistic creation becomes the paradigm mode in which people can come to self-definition” (p. 62).

Now, the success of dialogical relations with others requires many things. Among others, it requires the *recognition of difference*: the recognition of different ways of being. This is Taylor's alternative to the conformist view of relativism and its inarticulable mellow posture induced by individualism.

To come together on a mutual recognition of difference . . . requires that we share more than a belief in this principle; we have to share also some standards of value on which the identities concerned check out as equal. There must be some substantive agreement on value [and this] requires a horizon of significance, in this case a shared one. (Taylor, 2003, p. 52)

And this, Taylor thinks, can remove the aforementioned instrumental outlook of our authentic relations to others and make them truly identity-forming in a way that makes room for the other to be, at least to some extent, constitutive of the self.

To sum up, through his ethics of authenticity, Taylor suggests a conception of the self as a communal aesthetic creation. The ethics of authenticity helps us move beyond the confines of individualism where "there is nothing there beyond the self to explore" (2003, p. 90) and its ethical corollaries of an inarticulable ethical debate. This inarticulable debate gets pushed to the sidelines of society and ends up lost in the comfortable traps of relativism that only help to maintain a societal status quo where marginalized people, minorities, the poor, and all those who were supposed to benefit from the promises of the modern world are left to themselves.

Of course, we could argue that capitalism is so solidly entrenched today that there is no way to overcome it. The third point of the ethics of authenticity is a plea to reconsider this pessimistic view, a view that could be seen as a symptom of the alienation that capitalist societies inflict upon its citizens (Taylor, 2003, p. 116), a point also made by Freire (2005). In Taylor's account, our dialogical reasonings *can* make a difference. "It is still the case that [in society] there are many points of resistance, and that these are constantly being generated" (Taylor, 2003, p. 99). We certainly can generate a common consciousness: "The predicament alters when there comes to be a common consciousness . . . a common understanding" (pp. 100–101). For "Successful common action can bring a sense of empowerment and also strengthen identification with the political community" (p. 118).

Dialectical Materialism

We have seen that in the ethics of authenticity, the individual is conceived of as creating itself inwardly, in a dialogical reasoning with others, discovering, against the background of a social horizon of significance, an original identity in it, and attending to the originality that makes it irreplaceable, unique. Drawing on a dialectical materialist perspective, Balibar tackles the problem from the opposite side, so to speak. For him, indeed, the identity of the self is fundamentally ambiguous: "*there is no identity which is 'self-identical'*" (2002, p. 57; italics in the original). There is, hence, nothing in our interiority against which to check how we are doing

in our endeavors of self-discovery. Like Taylor, though, he thinks that the self is constituted in the social plane. But he opts for a view that parts from the kind of essentialism that still seems to haunt the ethics of authenticity.

The Non-essence of Essence

Balibar follows in the footsteps of Marx's fundamental break with the speculative essentialist tradition that locates the self inwardly, in a kind of interior space. Marx's break occurs in the sixth thesis on Feuerbach (Marx, 1998). In this thesis, Marx borrows the terminology of the idealist and the materialist traditions and still talks about essence, but in a way that puts it upside down. Marx writes, "the essence of man [sic] is no abstraction inherent in each single individual. In its reality it is the ensemble of the social relations" (Marx, 1998, p. 570). Marx argues that, in the idealist and materialist traditions, the understanding of the essence of the individuals is the result of a defective process of abstraction. It is a defective abstraction that results from separating the individuals from precisely that which gives them their *effective reality* (Wirklichkeit) or effectiveness. What these traditions fail to see is that the individuals' effective reality is "not simply a 'reality,' in the sense of a factual existence" (Balibar, 2014, p. 210); it is rather a cultural-historical reality in which their human process of realization unfolds.

Modernity has followed the idealist and materialist traditions and has pictured the self as a representative of something stable, static, a type, a same type, conceived separately from real life—reality. This self is "formed (or 'created') *according* to its essential properties" (Balibar, 2014, p. 213; emphasis in the original), which already reside *in* it.

It is therefore only a posteriori, when they are already constituted as [generic as well as unique] individuals, that they can *relate to each other* in different ways, but these relationships are by definition accidental, they do not define their essence. (Balibar, 2014, p. 213; emphasis in the original)

Where does the idea of the allegedly essential properties of the individual come from? Following de Libera's (2016a, b) archeology of the subject, Balibar, investigates the basis underpinning the idea of interiority as the inward site where our originality, our real essence, would be lodged. One of its components finds inspiration in "the postaugustinian model of the subjectivation of the individuals through the singular relationship they have with their creator" (Balibar, 2014, p. 212). This relationship with the creator is *internal*. As Augustine claimed, "Do not go outward; return within yourself. In the inward man [sic] dwells truth" (cited in Taylor, 1989, p. 129).²

² In a previous work, perhaps his best-known work, *Sources of the Self*, Taylor notes that "Augustine shifts the focus from the field of objects known to the activity itself of knowing; God is to be found here. This begins to account for his use of the language of inwardness" (Taylor, 1989, p.130).

In the path toward the secularization of the relation to the other mentioned in the Introduction of this chapter and witnessed by political treatises such as Jacobus de Cessolis's *Liber*, the Augustinian inner realm of truth, the road to God, becomes the road to the self. It is, then, from this tradition that Marx breaks in the sixth of his *Theses on Feuerbach*. The excerpt of the sixth thesis mentioned above has two parts: a negative part and a positive one. The negative part is the one that breaks with the Augustinian tradition: "the essence of man [sic] is no abstraction inherent in each single individual" (Marx, 1998, p. 570). The positive part is a kind of pun that targets idealism. In this part, Marx tells us what the genuine essence of the individuals is: "the ensemble of the social relations." (p. 570). In fact, the positive part tells us—if I can rephrase it this way—that the *essence* of the individuals is a *non-essence*.

Self and Other as an Ensemble of Social Relations

The conclusion we reached in the previous sections is that there is no such thing as an essence of the self or an essence of the other. The very fabric of the individual is made up of threads of *social relations* that the individual finds in her society. So, an understanding of self and other is not likely to succeed by inspecting an essence that presumably is lodged in the interiority of each one of them. An understanding of self and other requires investigating the *ensemble of social relations* that shape and organize their *effective reality* (Wirklichkeit, as opposed to an *abstract* reality). In fact, in Marx's analysis, the essence of the modern self—an essence conceived of as its distinctive, original trait—is but a transposition of the political and economic idea of *private property* to the anthropological realm: your essence is what you, and just you, own. Marx suggests that with the development of bourgeois life, private property "took the place of God as the 'inner truth' and source of sovereign injunction for man" [sic] (Balibar, 2014, p. 217).

We can see, then, the implications of this view in the conception of self and other. In the abstractions that run throughout materialism and idealism, the individuals are conceived of as *naturally separated from others*, encountering and entering in relation with the other a posteriori. What Marx suggests is the opposite movement: the social relations are already there, *before* we start moving into life. From an ontogenetic viewpoint, the primacy is not attributed to the self, nor to the other, but to the *social relations* that are already shaping and organizing human activity. The founding categories of an understanding of self and other are "the constituent social relations of the human being, and the praxis or movement of practical transformation already at work in every form of society" (Balibar, 2014, pp. 228–229). In this way, we can recognize the social determinations of self and other in their authenticity.

"The heart of Marx's invention," Balibar explains, "is not the suppression of the anthropological problem, but its transfer from the field of bourgeois metaphysical abstractions to that of concrete historical and social determinations" (2014, pp. 236–237).

Now, the determinations of self and other should not be understood in a causal sense. Such an interpretation has caused many misunderstandings: self and other become portrayed as being simply *produced* by a social setting (for an example of this interpretation in mathematics education, see, e.g., Cobb et al., 1997), as if the individuals were simple marionettes of their social context. This is not what determination means. The determination of self and other refers to the idea that self and other emerge jointly out of social relations in a dialectical movement through which they acquire specificity, purposiveness, distinctiveness, concreteness—a movement during which social relations are transformed and new ones are created (Radford, 2021a).

To understand this idea, we need to realize that the ensemble of social relations is something *fundamentally incomplete*, something always in *transformation*. Social relations are enacted by self and other in the movement of their *praxis* and, by enacting them, social relations change (they are contextualized, they are generalized, they are contested, subverted, etc.). As Balibar (2014, p. 233) notes, “The social relations that make up the human form an open network to which neither a conceptual closure ... nor a historical closure ... should be conferred.” In a similar vein, referring to the ensemble of social relations, Macherey (2008) notes that they are “‘brought together’ in the figure of a unity that is not simple, but complex” (p. 151). In Marx’s sense, social relations can only be understood as constituting

a multiplicity that cannot be totalized a priori because they form a “whole,” a totality of fact and not of law, which only holds together through the encounter of society’s various social relations, an encounter that is not inevitably harmonious or convergent, but which can and does take violent and conflictual forms. (Macherey, 2008, p. 151)

In the dialectical materialist perspective, social relations define self and other because these relations constitute them every moment in different manners. “They thus provide the only ‘effective’ content of the notion of human essence” (Macherey, 2008, p. 151). It is vain, hence, to seek in the abstract idea of “human species” or in the similarly abstract idea of “individual” the essence of the self. Such an “essence” can only reside in complexes of social relations: “this is what constitutes the effective essence of historical man [sic] who, far from being a primordial data of nature, produces himself by indefinitely recomposing these complexes” (Macherey, 2008, p. 152). In the end, we realize that “‘the human essence’ is only an illusion which masks and usurps the effectiveness of social relations, taken as a whole” (p. 153).

The Ontologizing of the Social

In this line of thinking, there is an ontologizing of the social: It is there, in society, that we can find the nature of self and other. Society is no longer seen as an ensemble of monads linked by distributed affections; nor is society reduced to contractual relations, as it appears conceptualized in Jacobus de Cessolis’s *Liber* or, more

explicitly, in John Locke's (1988) *Two Treatises on Government*.³ The ontologizing of the social invites us to see society as an open and unending dynamic system of activities organized around social relations always responding to, and shaped by, political, economic, historical, and cultural forces. These social relations continuously produce the similarities and differences out of which self and other come into existence (Radford, 2021a).

So, in contradistinction to Taylor, for Balibar, the constitution of the self occurs through an always problematic process of *identification* with others.

There is no *given* identity; there is only *identification*. That is to say, there is only ever an uneven process and precarious constructions, requiring symbolic guarantees of varying degrees of intensity. Identification comes from others, and always continues to depend on others. (Balibar, 2002, p. 67; italics in the original)

Identification is one of the poles of similarity. But similarity cannot exist without difference. In each social relation (e.g., the ubiquitous distribution of functions and division of labor), difference is built in. Social relations are determined as generators of similarities but also, and in a same degree, of differences, transformations, contradictions, and conflicts. "Differences create social relations" (Balibar, 2014, p. 238). And vice versa: social relations create differences. This is why the common being of any society includes heterogeneity.

If we understand society and their social relations as suggested above, and individuals as entities in economic, political, cultural, and historical transformations, the answer to the question "who am I?" cannot be a given singularity. For Balibar, the question "who am I?" is "rather the opening of a problem—with or without a solution: How can I (how can we, how can they) receive from another or a series of others (potentially infinite) *the objective sign of my (our, their) singularity?*" (1992, p. 15; emphasis in the original).

And we reach here, although from a different angle and with a different meaning, Taylor's idea of the necessary transindividual nature of self and other: the identity of the self, its individuality, is more than individual and other than individual: "It is immediately *transindividual*, made up of representations of 'us', or of the relation between self and other, which are formed in social relations, in daily life—public and private—activities" (Balibar, 2002, pp. 66–67).

In the dialectical materialist account, the constitution of self and other moves along the lines of an intrinsic and never solvable societal tension of centripetal and centrifugal forces, the former affected by a homogenizing movement of society (through its institutions, not just official, dominant institutions, like the school, but also revolutionary institutions) and the latter reflecting the infinite dispersal of identities that inhabit society.

So, the self, its identity, is not something to look for inwardly. Dialectical materialism invites us to envision the self not as a simple natural entity that, endowed

³ In Locke, the individuals relate to each other through consent and agreement to preserve their private property. According to Locke, private property is not sufficiently protected in the state of nature, "the original condition of all humanity" (Laslett, 1988, p. 98).

already with originality, moves *then* to the social and political arena to discover itself, but as a cultural-political entity in a continuous process of identification. This process of identification is at the same time a process of *differentiation*. The crux of the production of the self rests in this continuous and endless identification/differentiation process that is the mark of the “production of forms of human individuality in history—a process related to the always-already given transindividual ‘community’—by way of the complementary paths of resemblance and symbolic vocation” (Balibar, 2002, p. 71).

Self and Other in the Mathematics Classroom

If human individuals are social in nature, as Marx claims, how have they ended up understanding themselves as *a-* or *pre-*social beings? That is, how can individuals have come to conceive of themselves as naturally separated from each other and to imagine the social as something derivative?

Fischbach (2015) reminds us of a passage in the *German Ideology* where Marx (1998) notes that the ideas that people form reflect, consciously or unconsciously, their activities—activities of an economic, spiritual, aesthetic or any other nature. There is, in other words, a correlation between the activities of the individuals and the ideas that they build about the world and themselves.

This epistemological correlation between activity and thought that Marx alludes to was previously articulated by the Dutch philosopher Benedict of Spinoza (1989): material life and intellectual life are two elements that run in parallel, two things expressing the same phenomenon, one materially, the other ideally.⁴ Marx pushed Spinoza’s idea further and contended that the ideas that dominate in a given society are those of its dominant group. The answer to the previous questions should be sought, then, in the kind of life and the ensuing social relations that are privileged under the effect of dominant groups. In our contemporary capitalist societies, as we know, life revolves around commodities and the underpinning outlook of private property. We have become socially organized in terms of consumption. We live to consume not only food or other objects satisfying our needs for survival; we also consume news, music, blogs, the Internet, and a myriad of other unimaginable things.

The ideas we form of ourselves and how we relate to others (e.g., the ideas of individualism at the heart of Taylor’s malaises of modernity) spring, hence, not from thin air, but from the activities we carry out, on a daily basis, in our life. To use Spinoza’s terms, these ideas are no more, no less than the expression, on the side of thought, of our material life (e.g., the power of action we find in society, the social shaping and structuring of our power of activity, and the ensuing affections produced on our sentient bodies).

⁴ Proposition 7 of Part 2 of *Ethics* reads: “The order and connection of ideas is the same as the order and connection of things” (Spinoza, 1989, p. 82).

Talking about society and following the aforementioned Spinozist-Marxist line of thought, Fischbach notes that

When this [social] life is limited, limited in its development and possibilities for fulfillment and realization, then it is necessarily accompanied by truncated and inadequate thoughts and representations. When this life is separated, cut off from the material conditions of its own reproduction, then it is necessarily accompanied by abstract and general, that is, also inadequate, thoughts and representations. (Fischbach, 2015, p. 69)

Individualism is the name of the ideational counterpart of a truncated and limited form of life. In the language of the *German Ideology*, the *pre-* or *a-*social individualist outlook of self and other is the model of being that is imposed upon us by the current organization of society and its emphasis on consumption. Social relations—the “essence” of self and other—are currently defined not in terms of collaboration, but competition. If collaboration still finds some room, it is not a collaboration per se; it is a meager collaboration in terms of *self-interest*. And, of course, the school is not an exception.

A few years ago, I was invited to conduct a workshop for teachers. The meeting was organized by a school board in whose schools I had been conducting my field research. During the meeting, a teacher expressed skepticism about the benefits of group work in the mathematics classroom. He suggested that we conduct a study to prove that learning was better done in groups (i.e., by having students collaborate between, and discuss among, themselves) than individually (i.e., as in direct teaching). The teacher asked for a test like those done by the pharmaceutical industry to prove that one product is better than the other. For this teacher, collaboration could be worthwhile to consider for a student if and only if the student’s marks increase. He conceived of collaboration as a tool for self-advancement—like a tool of capital grow (Radford, 2020a).

In this respect, the problem for the mathematics classroom is to move away from the utilitarian forms of human collaboration it tends to promote. The problem is to find room for new, *non alienating*, forms of human collaboration that could lead to the creation of rich social relations between self and other. For this to happen, we need to envision, imagine, and experiment with new forms of classroom activity where teachers and students can find possibilities for genuine human fulfilment and realization (Radford, 2020b, 2020c). Such classroom activity, I want to suggest, could be oriented in ways to allow teachers and students to make the experience of authentic new forms of inclusiveness and democracy—democracy understood as “reclaiming the right of being in dignity [in] what one socially is and could potentially be on the grounds of one’s reality” (Valero et al., 2012, p. 3).

In the theory of objectification (Radford, 2021a), in our work with teachers and their students, we try to create new forms and spaces of collaboration through an emancipative reconceptualization of teachers and students (Radford, 2012, 2014) and teaching-learning activity. The emancipation of teachers and students can only be achieved through emancipative praxis—in this case teaching-learning praxes or activities. The emancipative nature of such praxes refers to the efforts that teachers and students make in order to overcome the utilitarian and technicist conceptions of

mathematics and the ensuing subjecting social relations that result from seeing and practicing mathematics as a *consumable product*. In our work, teaching-learning activity is reconceptualized in such a way that, laboring together, teachers and students *collectively* produce mathematical knowledge, and in doing, so they co-produce themselves against the always contested background of culture and history. Rather than being the acquisition of knowledge, learning appears here as the social collective process of coming into presence, the process of speaking to the other, hearing the other, working for the other, and finding one's realization in the realization and fulfilment of others. Teaching-learning activity becomes *joint labor*—the joint labor of teachers *and* students, where teachers and students work together, hand in hand, and come to co-position themselves in the classroom considered as a public space that is intrinsically political. Joint labor is the materialization of what we call a *communitarian ethics*, one revolving around responsibility, engagement, and the care for others (Radford, 2021a, 2021b).

Joint labor, however, does not amount to a pacific and boring process of exchange and interaction. Joint labor is the bearer of tensions and contradictions. It is the bearer of the societal contradictions that constitute the very substance of the social relations we all encounter in society and that form our own transindividual essence. From an education perspective, the point, then, is to make those relations intelligible, to make them objects of consciousness, recognizable in their cultural-historical causes, and to work together in the creation of new social relations that could be conducive to more fulfilling relationships between self and other.

Acknowledgments This article is a result of a research program funded by the Social Sciences and Humanities Research Council of Canada/Le conseil de recherches en sciences humaines du Canada (SSHRC/CRSH).

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