

CHAPTER 4

SENSED OBJECTS, SENSING SUBJECTS: EMBODIMENT FROM A DIALECTICAL MATERIALIST PERSPECTIVE

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Introduction

In *Anthropology from a Pragmatic Point of View*, Kant tells us that sensibility is comprised of two parts: sense and imagination. “Sense is the faculty of intuition in the presence of an object. Imagination is intuition without the presence of the object” (Kant, 1978, p. 40). He then divides the senses into two classes: *sensus vagus* and *sensus fixus*. To the first belong the sensations of warmth and cold; to the second belong sensations ascribed to a certain part of the body. In Kant’s count, there are five senses: (1) touch (*tactus*), (2) sight (*visus*), (3) hearing (*auditus*), (4) taste (*gustus*), and (5) smell (*olfactus*). The first three he considers as more objective than subjective,

that is, they contribute, as empirical intuition, more to the cognition of the exterior object, than they arouse the consciousness of the affected organ. Two, however, are more subjective than objective, that is, the idea obtained from them is more an idea of enjoyment, rather than the cognition of the external object. (Kant, 1978, p. 41)

Kant follows here a long-standing tradition that tackles the role of the senses in the epistemological problem of how much we, humans, know or can know about material and ideal objects. Far from old and obsolete, the problem has gained a tremendous impetus in the past decades in fields such as philosophy (Derrida, 2005; Nancy, 2008), animal and human psychology (Gómez, 2004; Zaporozhets, 2002a, 2002b), anthropology (Geurts, 2002; Le Breton, 2009), and sociology (Rancière, 2010), leading to a marked interest in the question of the body and embodiment—not only in those fields, but also in mathematics education (Arzarello, 2006; Edwards, Radford & Arzarello, 2009; Radford, 2014a; Radford, Arzarello, Edwards & Sabena, 2017; Nemirovsky & Borba, 2003).

In this chapter, my goal is to offer a dialectical materialist conception of embodiment. In the first part, I discuss an episode from a Grade 1 class where 6-7-year-old children were dealing with a task requiring the reproduction of the first four terms of a figural sequence and the production of the two following terms. In the second part, I discuss two main perspectives around embodiment, one stressing the materiality of the world, the other stressing the subjective dimension of sensuous experience. In the third part, I present a dialectical materialist conception of embodiment.

Reproducing and Expanding a Figural Sequence

The episode that I want to discuss here comes from a longitudinal study in which we followed a cohort of students from junior kindergarten to Grade 1. The episode in question is about the sequence shown in Figure 1.

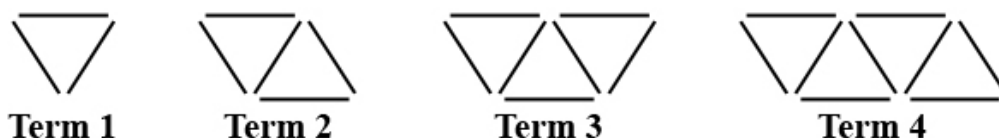


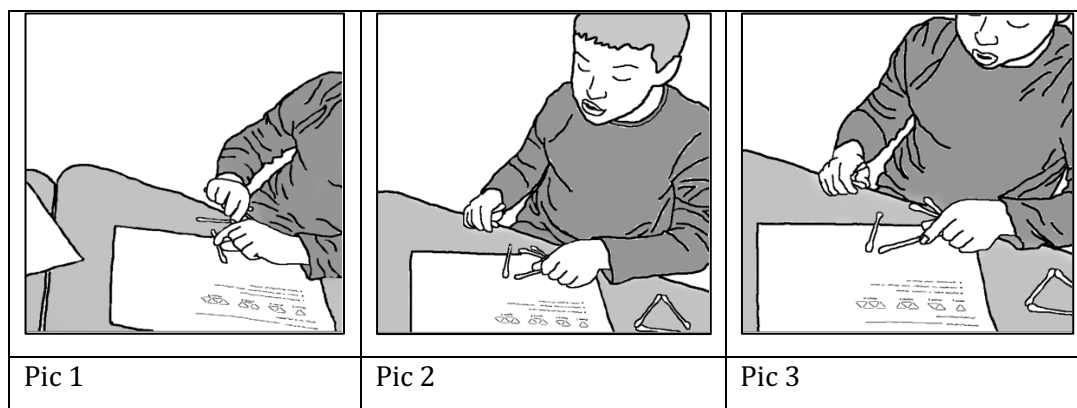
FIGURE 1. The first terms of a sequence explored in a Grade 1 class

In the first part of the lesson, the teacher provided the students with a bag of Q-tips and invited them to reproduce these terms. The reproduction of terms was followed by a question of generalization: the students were asked to *build* Term 5 and Term 6 of the sequence. In the last part of the lesson, the students were invited to draw with pencil Terms 5 and 6, and then to find out the number of Q-tips in Term 10, without drawing or making the term.

The task of reproducing the terms turned out to be quite difficult for the children. To reproduce something, this something—the “model”—has first to be endowed with some *meaning*. It is through this meaning that the process of reproducing the model can unfold. But to endow the model to be copied or reproduced with meaning, means first of all, that it has to become a *sign*. For, as Bakhtin (1986, p. 113) puts it, “only signs (including words) have meaning.” And meaning invokes a creative understanding: “Therefore,” Bakhtin continues, “any study of signs, regardless of the direction in which it may subsequently proceed, necessarily begins with understanding” (1986, p. 113).

Now, understanding is generally understood as something conceptual. This is not the sense that I want to use. Understanding, as I see it, is not something that comes immediately to the children’s consciousness. Understanding unfolds in a dialectical process where the embodied and material doing of the children and the children’s embodied and material reflection of the done and what is still to be done affect each other. In the present case, understanding unfolds through the children’s experimentation with the Q-tips, when they move them on the desk with their hands and, at the same time, see and assess the emerging form.

Thus, when Nicolas starts building Term 2, he grabs six Q-tips and puts three in each hand. He places one of the Q-tips on the desk as if it were the right side of a triangle, then adds another one as if it were the base (as seen from the child’s perspective; see Figure 2, Pic 1). He adjusts the position of the two Q-tips, but he does not seem to like it. He then proceeds to remove the second Q-tip, places the Q-tip at the left side of the triangle (see Pic 2), adjusts their positions (see Pic 3), then adds the base (see Pic 4). He complains that the reproduction looks “weird.” He has three Q-tips in his hands, returns one of them to the heap of Q-tips on the table and, when he is about to start adding Q-tips on the table to finish making Term 2, he realizes that he does not have enough room. He moves the form up, the form collapses (Pic 5), he repairs it and adds two Q-tips. The reproduced Term 1 and Term 2 look as shown in Pic 6.



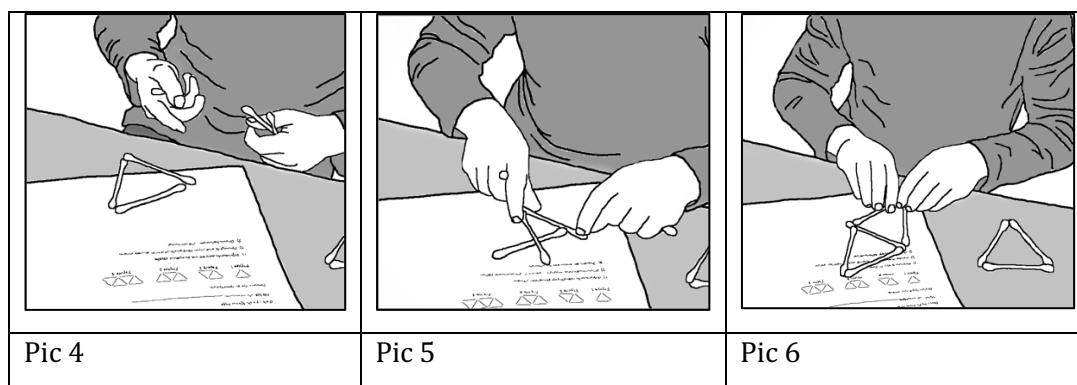


FIGURE 2. Nicolas building Term 2

As we can see in Pic 6, Terms 1 and 2 do not conform to their relative position in the model. It seems that Nicolas, like many other children, focuses on the terms as such. The spatial and numerical relationality *between* terms does not seem to be in the child's focus of attention—yet. This point is even more evident when he starts reproducing Term 3.

He starts saying “Term 3.” Then, saying “one, two” he takes two Q-tips in his hands. As he places the Q-tips on the desk, he says “one, two” building the right side of the form, then the left side (see Figure 3, Pic 1). Then, after picking up another Q-tip, he builds the base. He obtains a first triangle (see Pic 2). He grabs two more Q-tips and adds them almost simultaneously to the *right* of the triangle, obtaining a form with two triangles (see Pic 3). The first part of the form—a two-triangle form—has been quickly accomplished as compared to the time Nicolas spent in Term 2. He picks up two more Q-tips and tries unsuccessfully to add them to the *left* side of the form (see Figure 3, Pic 4) (as seen from the child's perspective). He spends eight seconds (which is an extremely long period of time here) looking at the form trying to figure out where to put the Q-tips. He realizes that Term 2 is too close to continue the construction of Term 3. He decides to move Term 2 out of the way (Pic 5). With more room, he is able to add the two Q-tips to the *left* of the emerging form and, with some difficulties, he finishes Term 3 (Pic 6).

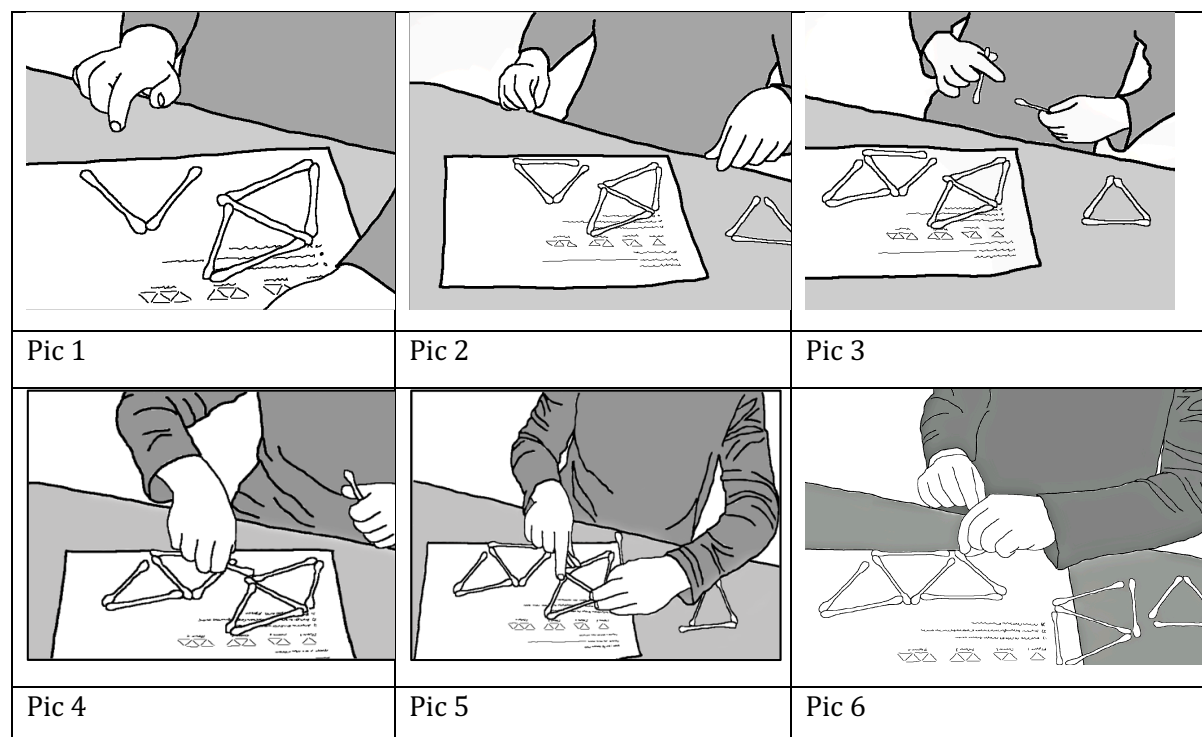


FIGURE 3. Nicolas building Term 3

As we can see, the reproduction of forms requires the children to make sense not only of the forms themselves, but of their place in the space. To make sense, this space needs some references—some *anchoring reference points*. The consideration of these spatial references—references that following linguistics we may call *spatial deictics* (Nyckees, 1998), as they index and thus make *distinctions* in space—allows for the emergence of relations between objects. What Nicolas’s deeds suggest is that this relation is not taken into consideration yet. But this is not to say that there are no special deictics at all. There is an *order*: Nicolas produces the terms from left to right, as in the model. This emergent spatiality results from a relationship *between* the child and the term. It is not an inter-figural relationship. In other words, it is first of all mediated by the child’s body. This is why the position of the forms on the desk is not anticipated from the start but is produced on the go as new forms appear on the desk. There is no relational reading of the terms in the model, a reading that would allow the children to imagine their positions before starting the reproduction process. No wonder that, still holding the Q-tips in his hand and searching for the right place for them in the in-progress Term 3 (see Figure 3, Pic 2), Nicolas asks the teacher, “Why is it so hard?”

The spatiality of the terms is one of the problems—a fundamental problem, but not the only one. The other fundamental problem is the *construction* of the terms. The model does not provide an indication of *how* the terms should be reproduced. The model is silent in terms of the *order* of construction. In the model, the terms appear *in toto*, as a finished whole. The children have then to come up with the idea of a sequence of actions whose most essential feature is to apply not only to one term or even to a few terms, but to *all* terms. Kant conceived of this idea as a mental “universal procedure of imagination” (Kant, 2003, p. 182, A140/B179). He called this universal procedure the *schema* of the concept—in this case the *rule* or *procedure* that allows the children to construct the terms of the sequence. Kant considered imagination as a human built-in faculty in charge of ensuring the synthesis of sensible impressions. In Kant’s account, hence, Nicolas’s sensible kinesthetic actions of placing the Q-tips on the desk would provide him with diverse sensible impressions. These sensible impressions would be collected by the intellectual imaginative faculty that Nicolas allegedly already possessed at birth. With the help of this rule-based imaginative faculty, “which precedes all the data of intuition” (Kant, cited in Eisler, 1994, p. 41), Nicolas would carry out a synthesis of the sensible diverse, transforming them into something conceptual (Kant, 2003). Without the participation of the intellectual imaginative faculty, regardless of how hard and how long Nicolas would have looked at the lines and forms, he would have not been able to even reproduce Term 1 of the sequence. Indeed, without the help of this imaginative faculty, Nicolas would have been incapable of discerning lines and forms. He would see pure noise—*pure chaos*. Now, how, despite possessing the a priori intellectual faculty of imagination, could Nicolas find it difficult to reproduce the first terms of the sequence? Kant would answer that Nicolas’s troubles had to do not with the faculty of imagination but with the *faculty of judgment*; that is, the faculty of distinguishing the particulars that fall under the rules of the faculty of imagination. This faculty, Kant argued, “cannot be taught, but only exercised” and learned through experience (1978, p. 93). Nicolas’s problem, in this account, is that he has not yet learned to subsume the particular material Q-tips under the general form.

Piaget dismissed Kant’s a prioristic stance and considered the schema as a generalizable and organizing feature of an action or a sequence of actions in the construction of a concept. “The scheme of an action is, by definition, the structured group of the generalizable characteristics of this action, that is, those which allow the repetition of the same action or its application to a new content” (Beth & Piaget, 1966, p. 235).

In Piaget’s perspective, the first schemas that children build are the sensorimotor schemas of physical actions, which are later on interiorized or internalized, becoming thereby *mental constructs*. “In the case of those actions which will become interiorised in the form of operations, the schemes of actions then include their most general characteristics, that is to say,

the characteristics of co-ordination as such” (Beth & Piaget, 1966, p. 235). While placing Q-tips on the desk, Nicolas would hence have formed a schema; he would have internalized it at a certain point in time, and from there he would have become capable of producing Term 2, Term 3, and so on.

Piaget’s approach has been criticized on several occasions (see, e.g., Buck-Morss, 1975; Otte, 1998; Rotman, 1978; Wartofsky, 1983). I will limit myself here to mentioning that Piaget’s work oversimplifies the role of material culture, the body, and the senses in the process of knowing. Material culture is reduced to mere stuff that offers itself to the agent’s action in a transparent and cognitively neutral way. The body and the senses appear as a lower form of cognition from where emerge more theoretical levels of cognition up to the formal level of logical-mathematical cognitive structures. Of course, Piaget is not alone on this path. The reductionist view of material culture and the oversimplified conception of the senses and the body is part of a long-lasting tradition in Western thought. For instance, when Husserl turns to sensation, he starts by making a distinction between sensibility and reason: “We distinguish here,” he says “*sensibility* and (let us say) *reason*. In sensibility we distinguish *primal sensibility*, which does not contain any sediment of reason, and secondary sensibility, which arises through a production of reason” (Husserl, 1989, p. 345; emphasis in the original). In a posthumous work, Husserl speaks of “Individual objects, spatial things, [as] constituted by ‘apprehension,’ ‘apperception,’ of sense data” (1973, p. 255). In this account, the sense data are submitted to a constitutive synthesis of consciousness that, in return, bestows meaning to that data, constituting thereby the raw material upon which are founded “the specific forms of judgment encountered on the level of formal logic” (Churchill, 1973, p. xviii; see also, Hopp, 2008).

At its most general level, the enduring dichotomy between the senses and reason and the conception of the senses as a lower form of cognition rests on a simplification that reflects the distinction between manual and intellectual labor. This distinction has been the chief characteristic of social formations of an aristocratic nature in antiquity (Levi, 1974) up to the most recent capitalist formations (Fischbach, 2015). It has had a direct impact on our understandings of the world, the human mind, and the human more generally. In this understanding of the human, the body and the senses are considered a hindrance, or, as Plato put it in his dialogue *Phaedo*, “an impediment which by its presence prevents the soul from attaining to truth and clear thinking” (Plato, 1961, 65e–66a, p. 48). In more indulgent accounts, the body and the senses are considered as an ephemeral moment in the trajectory of the mind in its path towards the conceptual realm.

In the rest of this chapter, I want to explore a different conception of embodiment—one that seeks to give room for a cultural-historical understanding of the senses and the body in knowing and becoming. I start by commenting on some recent efforts that have been made to reposition the epistemic dimension of the body.

Movement

Maxine Sheets-Johnstone is one of the contemporary scholars who, in trying to overcome the pitfalls of rationalism, has emphasized the epistemic role of the body. Her approach to the body is articulated around the central concept of *movement*. She argues that “Spontaneous movement is the constitutive source of agency, of subjecthood, of selfhood, the dynamic core of our sense of ourselves as agents, subjects, selves” (2011, p. 119). She goes on to remark that “it is odd that philosophers would overlook the *sui generis* character of movement and fail to explore its significance” (2011, p. 117). For, in Sheets-Johnstone’s line of thought, we humans are first of all “animated organisms”—a term that she borrows from Husserl (1980, p. 94). She notes that

[A]nimation is at the core of every creature's engagement with the world because it is in and through movement that the life of every creature . . . acquires reality . . . In the beginning, we are simply infused with movement—not merely with a *propensity* to move, but with the real thing. This primal animateness, this original kinetic spontaneity that infuses our being and defines our aliveness, is our point of departure for living in the world and making sense of it. It is the epistemological foundation of our learning to move ourselves with respect to objects, and thus the foundation of a developing repertoire of “I cans” with respect to both the natural and artifactual array of objects that happen to surround us as individuals in our particular worlds. (2011, p. 117)

The discovery of what a body can do is not the result of a contemplative rationalist cognition. It is as a *result* of the body's movement that the body discovers a “realm of sheer kinetic ‘I cans’: I can stretch, I can twist, I can reach, I can turn over, and so on” and that the body discovers an “open-ended realm of possibilities” (2011, p. 117). It is from movement that “kinesthetic consciousness” comes into life and “in turn defines an emergent, progressively expanding consciousness” (2011, p. 117). It is here, in movement, where we find indeed “the beginning of cognition” (2011, p. 118). For “Our first cognitive steps are taken by way of our own movement” (2011, p. 118).

If we return to our example, we see Nicolas taking six Q-tips from the heap of Q-tips lying on the desk. Nicolas is not anticipating first where and how he is going to place the Q-tips he grabbed to reproduce the second term of the sequence. In taking the Q-tips he is at the same time reflecting where and how to put them. Reflection does not precede action. There is a primal sheer kinetic “I can” that starts unfolding and coming into life in the movement of Nicolas's hands and body where, to use Sheets-Johnstone's term, “kinesthetic consciousness” is arising.

The awareness of corporeal powers does not (and could not) arise *ex nihilo*. It arises from tactile-kinesthetic activity: chewing, reaching, grasping, kicking, etc. The awareness of corporeal powers is thus not the result of reflective musings, whether with or without language. It is not a matter of wondering, What can I do? On the contrary, the sense of corporeal powers is the result either of moving or of already having moved. (Sheets-Johnstone, 1990, p. 29)

The example of Nicolas making Term 3 allows us to see this point more clearly. To build Term 3, Nicolas started building the first two triangles of it (see Pics 1-3 in Figure 3 above). It is only when he tries to build the third triangle that he realizes that he does not have room left to continue (see Pic 4 in Figure 3 above). “In discovering ourselves in movement and in turn expanding our kinetic repertoire of ‘I cans,’ we embark on a lifelong journey of sense-making” (Sheets-Johnstone, 2011, p. 118). In other words, “I can do” is the result of a primal “I can” that is embodied in Nicolas's deeds/reflections.

Sheets-Johnstone's work has the great merit of making us rethink the role of the body in our accounts of knowing. It is part of a recent impressive amount of scholarly efforts that have led us to what she calls the “corporeal turn” (Sheets-Johnstone, 2009; see also Steitz, 2000; for a literature overview see Wilson, 2002). At the same time, Sheets-Johnstone's account shares with most of the current perspectives on embodiment the classical assumptions of the senses and the body as unproblematical *givens*. It joins the uncountable perspectives that take the subject as a *constituting subject*: a body that appears as the locus of meaning and conceptuality, a body that, in its movement, creates “corporeal concepts” (Sheets-Johnstone, 1990, p. 29) and “corporeal consciousness” (2011, p. 48). For in this account, as in all subjectivist approaches to embodiment and the human mind more generally, it is the individual who, through her body, constitutes consciousness and the world. It is, indeed, this conception of the individual and the

body that allows Sheets-Johnstone to assert that “kinesthetic consciousness” is a “world-constituting consciousness” (2011, p. 128). This is why here Piaget is not found faulty. Piaget was not wrong. His problem was that he was not as radical as he should have been. It was not human action that was the creator of meaning, conceptuality, and selfhood. It was *movement*—and that he missed.

The problem with Sheets-Johnstone’s theoretical position, as well as with all subjectivist positions, is that it focuses on the individual alone. It forgets that individuals come to create concepts in a cultural-historical world that, before the body moves, presents the body with affordances and limits. The “I cans,” indeed, from where concepts and conceptions of selfhood and agency arise, always unfold within webs of economic, political, social, cultural, and historical structures that shape and profoundly affect our movements in the world and what we can and cannot do. The only beings who could move as they wish, freely and exempt of any prior structuring possibilities, are those mythical beings that preceded culture formations, like the gods and goddesses of Homer’s time.

Sensuous Cultural-Historical Cognition

In the rest of this chapter I would like to explore a different path to the problem of embodiment, with an interest in teaching and learning (for other recent explored paths toward embodiment in teaching and learning see de Freitas (2016); see also Edwards, Radford, and Arzarello (2009); for an overview see Radford, Arzarello, Edwards, and Sabena (2017)). My starting point is a theoretical position according to which the subject and the body are *not* considered as the *constituting entities* of meaning and conceptuality. I want to move away from the subjectivist position featured in the work of Kant, Piaget, Sheets-Johnstone, and many others.

The subjectivist position has been the cornerstone of Western philosophy and for which Husserl presents us with the insightful metaphor of the “ray emanating out of the Ego” (Husserl, 1989, p. 346). The Ego is the *origin* of the ray. The ray connects the Ego to the world and constitutes thereby “the genuine ‘intention toward’” the world (Husserl, 1989, p. 347). In his late work Husserl made a tremendous effort to move beyond such a subjectivist position. Indeed, in his late work he became interested in questions of inter-subjectivity and in the *nature* of the world that is offered to us in perception; that is, quotidian *life-world*. He came to realize that life-worlds, different from one culture to another, set the standards that delineate a world-horizon of experiences. Sense data were no longer to be interpreted by a solipsist Ego, but by a social Ego that lives in a cultural life-world with its own historical truths.

[W]hen we are thrown into an alien social sphere, that of the Negroes in the Congo, Chinese peasants, etc., we discover that their truths, the facts that for them are fixed generally verified or verifiable, are by no means the same as ours. (Husserl, 1970, p. 139)

Yet, he still maintained that there was also a “general structure” (Husserl, 1970, p. 139) that is “not itself relative” (p. 139)—a general structure that is valid for all subjects and cultures (Europeans, Chinese, Hindus, etc.). In Husserl’s account, this general common structure allows us to overcome the “embarrassment” (p. 139) of relativism and provides us with the ground for the more objective world of science. In doing so, Husserl’s work provides us with an outstanding historical example of an agonizing effort to move away from the long-standing subjectivist position that, in the previous section, we see reappear in Sheets-Johnstone’s approach. Husserl sought to move to a broader theoretical position where the conceptual realm is more than the result of the sensing agent in moving, perceiving, sensing, interpreting, and making judgments about objects in the world. Colossal as it is, Husserl’s work, however, remained committed to a scientific transcendental view of truth (Derrida, 1989), where cultures provide the basic substratum for a unique theoretical stance to emerge, namely Western science. Western science is the name of this objective endpoint where the disparate cultural array of sense data converges.

Briefly returning to my point, what I want to explore is an account of embodiment that is something different from the ones that posit the Ego as “the center of a surrounding world appearing to him” (Husserl, 1989, p. 358) and the ones that posit the Ego’s movement as the source of agency and conceptuality (Sheets-Johnstone, 2011, p. 128). But I rush to say that my intention is not to obliterate the subject, the body, the Ego. If my effort is an effort to overcome the primacy of the subject, it is also an effort to overcome the primacy of the object, the world. I draw on Marx’s materialism, which, epistemologically speaking, is in fact an effort to rethink the relationship between the object and the subject as mutually constitutive. And that which mutually constitutes both is sensible activity, *praxis*.

Marx started articulating this idea, which underpins in a central manner his concept of materialism, in the *Theses on Feuerbach*. Ludwig Feuerbach was a Hegelian converted to empiricism. In the *Philosophy of the Future*, Feuerbach contended that

The real in its reality or taken as real is the real as an object of the senses; it is the sensuous. Truth, reality, and sensation are identical . . . Only through the senses, and not through thought for itself, is an object given in a true sense. (Feuerbach, 1966, p. 51)

The *Theses on Feuerbach* is an ensemble of eleven propositions that Marx wrote in a notebook. They are not a finished product; they are rather sketchy notes intended perhaps as a kind of aide-mémoire. The first sentence of the first thesis reads as follows:

The chief defect of all previous materialism—that of Feuerbach included—is that things [*Gegenstand*], reality, sensuousness are conceived only in the form of the *object*, or of *contemplation*, but not *as human sensuous activity, praxis*, not subjectively. (Marx, 1998, p. 572; translation slightly emended)

In this sentence, Marx presents a critique against Feuerbach’s materialism. The problem with such materialism is that, in emphasizing the object, it is incapable of grasping things, reality, and sensuousness from the viewpoint of the *subject*; that is, subjectively. Feuerbach’s materialism *separates* the subject and the object. Here, truth lies on the side of the object. In Feuerbach’s materialism, cultural objects, such as objects of production, for instance, appear as objects *as such*. This kind of materialism fails to grasp these objects subjectively, as objects of consumption.

The previous sentence is followed by the following sentence, where the critique is addressed to idealism:

Hence it happened that the *active* side, in contradistinction to materialism, was set forth by idealism—but only abstractly, since, of course, idealism does not know real, sensuous activity as such. (Marx, 1998, p. 572)

In this passage, Marx says that the “*active* side,” that is to say, the side of an active individual, is taken into consideration in the idealist account, but in an *abstract* manner only. The individual appears in an abstract manner, for the individual is considered as acting freely, according to her whims and her natural powers. In doing so, the individual is converted into a corporeal world-constituting consciousness, to borrow Sheets-Johnstone’s expression. Here, truth lies on the side of the individual. The objects of consumption, to continue with our example, are posited as merely subjective, unhistorical, independent of the cultural production that subsumes them.

As we can see, idealism makes the same mistake as Feuerbach’s materialism: it also *separates* the subject and the object. While idealism puts the emphasis on the subject, Feuerbach’s materialism puts the emphasis on the object. For Marx, things, reality,

sensuousness as conceived by idealism, Feuerbach's materialism, and previous materialisms (e.g., Democritus's) end up in a mutilated materialism

to which we must oppose a complete materialism, that is to say a materialism that restores the broken link between subject and object—a materialism which requires to bring forward the concept of . . . *praxis*, which combines both aspects. Only this concept is indeed able to make the connection between an object, a determinate element of natural reality, and a subject actively engaged in the work of transformation of the world. (Macherey, 2008, p. 40)

Praxis or activity (*Tätigkeit*, in Marx's German terminology) is not something abstract taking place with abstract individuals (epistemic subjects, cognitive subjects, etc.). Activity in the sense of *Tätigkeit* takes place with flesh-and-blood individuals in their *practical* and *sensuous* reality. Instead of merely unfolding *in situ*, speculatively, as something occurring naturally, out of the blue, activity (*Tätigkeit*) is, on the contrary, a *historical* process. Activity, taken in this sense, follows indeed "a movement that is not only natural but historical (Macherey, 2008, p. 41). "Instead of conceiving of the world as an objective whole given in a natural manner," human activity accomplishes "the concrete fusion of the subjective and the objective" (p. 41).

Marx's concept of activity or *praxis* is an explicit effort to overcome both: (1) speculative idealism that endows consciousness with false constitutive powers, and (2) the false concrete materialist sensualism of Feuerbach that conceives of the senses and the concrete in an abstract manner, as if the senses were strict natural entities of the order of nature and nature alone. In this way, the first of the *Theses on Feuerbach* achieves an inversion of perspective moving the conception of the senses and reality from heaven to earth. It does so by replacing the speculative stance of idealism and materialism with the idea that object, reality, and sensuousness are entities that can only be understood against the backdrop of practical human activity; that is to say, human historical labor. We move here from a view of the senses, the body, and the material world in which the senses and the body are conceived of as natural given to a view according to which they are *produced* historically and culturally.

Referring to materialism in general, Fischbach notes that, "The sensible material on which materialists have based themselves (both sensible things and human faculties, human sensibility) has been taken as a de facto datum and not as the sensible product of a human activity itself sensible" (Fischbach, 2015, p. 31). Explaining the meaning that human sensible faculties acquire in Marx's account, Fischbach continues:

[Human] sensible faculties such as hearing or sight are not given identical to themselves from all eternity . . . they are produced, they result from and are modified by human activity, they participate in this activity and are an aspect of it. (Fischbach, 2015, p. 31)

To understand Marx's materialism, we need to bear in mind that for him the human is part of nature. And so are the human senses. Of course, Marx is far from original in this claim, which was central to Spinoza's (1989) view of the world. Husserl, for instance, contends that "human beings can also be regarded as reality or nature" (Husserl, 1989, p. 351). This idea has survived in several guises through centuries, reappearing in contemporary discourses and conceptions of the body, as Sheets-Johnstone's work shows.

For Marx, as for Husserl, Sheets-Johnstone, and many others, there is something fundamentally basic and vital in being human: the capacity of self-activation (*Selbstbetätigung*). The difference is that Marx suggests a different view of the human, one in which, through activity, nature—the senses included—becomes humanized nature. The senses and their cultural, historical, and political development are entangled with the objects they produce and attend to, not as transcendental objects or as merely objects of touch and perception, but as

objects of human activity. By encountering these historical-cultural objects created by previous generations, humans create human sensibility and in doing so, humans express and confirm what Marx calls the human essential powers:

Only through the objectively unfolded richness of man's essential being is the richness of subjective *human sensibility* (a musical ear, an eye for beauty of form—in short, *senses* capable of human gratifications, senses confirming themselves as essential powers of *man*) either cultivated or brought into being. For not only the five senses but also the so-called mental senses—the practical senses (will, love, etc.)—in a word, *human* sense—the humanness of the senses—comes to be by virtue of its object, by virtue of *humanized nature*. The *forming* of the five senses is a labor of humanized nature. The *forming* of the five senses is a labor of the entire history of the world down to the present. (Marx, 1988, pp. 108-109; emphasis in the original)

Usually, Feuerbach and the materialists before and after him adopt, one way or the other, a conception of the world that is either already there for the body to touch. Likewise, idealists before and after him adopt a conception of the world as something constructed anew, subjectively, in the deployment of the “I cans” with which the body has been biologically endowed. So, the world is considered as an untouched world that is already there, neutrally, or that is being constructed on the spot, out of the blue, having, in one case as in the other, escaped the effects of history, politics, and economics to provide the body with an infinite array of experiences of movement. As Sheets-Johnstone puts it, “I can stretch, I can twist, I can reach, I can turn over, and so on” (2011, p. 117).

In *The German Ideology*, written shortly after the sketchy *Theses on Feuerbach*, Marx reproached Feuerbach for having missed the historicity of the sensuous world. Feuerbach, Marx contended

does not see that the sensuous world around him is not a thing given direct from all eternity, remaining ever the same, but the product of industry and of the state of society; and, indeed, [a product] in the sense that it is an historical product, the result of the activity of a whole succession of generations... Even the objects of the simplest “sensuous certainty” are only given him through social development, industry and commercial intercourse. (Marx 1998, p. 45)

What Feuerbach does not grasp, Marx argues, is that what we access through our senses is not the world in its intact materiality—Adam and Eve's intact world—but a world of labor replete with historical and cultural objects and sensuous activity:

The cherry-tree, like almost all fruit-trees, was, as is well known, only a few centuries ago transplanted by *commerce* into our zone, and therefore only by this action of a definite society in a definite age it has become “sensuous certainty” for Feuerbach. (Marx 1998, p. 45; emphasis in the original)

Cultural-historical activity dealing with nature, human objects, and reality, is hence the link between the subjective and the objective, between subject and object, and the ground where human senses are formed. With these ideas in mind, let us turn back to Nicolas and the reproduction of forms.

Reproducing Term 4

Nicolas starts building Term 4. He builds two triangles connected by a common vertex (see Figure 4, Pic 1). He looks at the model and in a discouraged tone says, "It is impossible!" Krista, his teammate, has just finished Term 3 and is working on building Term 4 (Pic 2). Nicolas takes a Q-tip, puts it close to the upmost right corner of the term he has built, moves it as if experimenting with various positions, and decides to put the Q-tip in an upward position (Pic 3). He adds a second Q-tip and says again, "It's impossible!" (Pic 4). He adds another Q-tip to finish the triangle. Unsatisfied with the form, he grabs all the Q-tips and says, "I am not capable of making the fourth."

A few moments later he is still complaining:

1. Nicolas: No. It is impossible, I cannot do it.
2. Krista: Why do you say that it is impossible?
3. Nicolas: I cannot do it . . . Oh my God, it's very hard. How do you do that?

The passage from constructing the first three terms to constructing Term 4 has proven to be very difficult for Nicolas. It is not enough to have a model in the perceptual field to reproduce it. To reproduce something or to imitate something is a task harder than usually assumed. In his remarks on imitation, Vygotsky noted that

Rooted in traditional psychology, as well as in the everyday consciousness, is a view of imitation that assumes it is a purely mechanical activity. From this perspective, a solution that is not reached independently is not considered an index or symptom of the development of the child's intellect. It is assumed that the child can imitate anything. What I can do by imitation says nothing about my own mind. It cannot be used in assessing development. This view is false. (Vygotsky, 1987, p. 209)

A certain kind of potential readiness is required to successfully imitate or reproduce something. As mentioned above, the form to be reproduced has to be endowed with meaning; and endowing something with meaning is a semiotic process that is far from self-evident. Certainly, Nicolas *sees* in the model some lines and forms, as he shows in the reproduction of the first three terms, even if the relative position of the forms or terms of the sequence is not taken into consideration to its full extent. But a more general relationship between lines and forms needs still to become apparent to his consciousness.

Krista's strategy consists in adding triangles. She starts with an inverted delta (Δ) triangle. She adds to the right of the first triangle a delta triangle, starting with the oblique sides.

4. Krista: (*While adding the base of the second triangle, she says*) It's easy!
5. Nicolas: It is not!
6. Krista: Yes, it *is* (*starting the construction of the third triangle, she places a Q-tip horizontally on top of the emerging inverted delta (Δ) triangle*).
7. Nicolas: No. It is not!
8. Krista: (*Talking to herself*) Like this (*placing a Q-tip diagonally to the right, finishing the inverted delta triangle*).
9. Nicolas: No. It is not.
10. Krista: (*Looking at the form that shows three triangles*) I am almost done!

Right after, Krista starts building the last part of the form. She moves her body to the front to better see the model, adds the left side of the last triangle directly beside the right side of the previous triangle (Figure 4, Pic 5), adds the right side, and then adds the base. Joyfully she exclaims, "Finished!" We can see that her Term 4 (shown on Figure 4, Pic 6) has some double Q-tips.

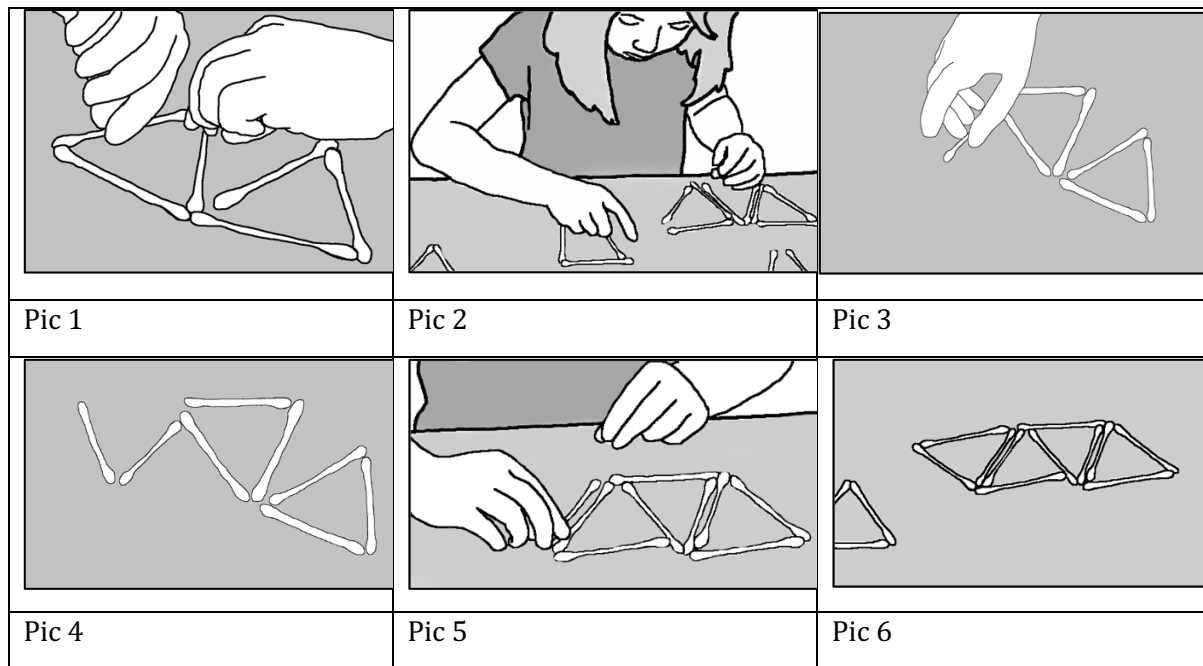


FIGURE 4. The construction of Term 4. Pic 1: Nicolas starts by building two inverted delta (Δ) triangles connected by a common vertex. Pic 2: Krista starts building Term 4. Pics 3 and 4: Nicolas tries to continue the construction of Term 4. Pic 5: Krista starts adding a triangle. Pic 6: Krista's final Term 4.

Neither Krista nor Nicolas has mentioned the term "triangle." Yet, their actions suggest that in their reproduction of the terms triangular forms are recognized. This is suggested in Nicolas's partial form shown in Figure 4 Pic. 1, and in Krista's Pics 2, 5, and 6. Furthermore, when Krista has finished building the three first triangular forms of Term 4, she pauses to count. She looks attentively at the model, then counts aloud the triangular Q-tip forms while pointing to them. In line 10 she utters, "I am almost done!"

Krista's building procedure can be summarized as follows: First, she builds an inverted Δ triangle. Second, she adds a whole three-Q-tip triangle to the right. Third, she adds two Q-tips to complete the form. Fourth, she adds a whole three-Q-tip triangle to the right. The result is that some segments in Term 4 appear as having two Q-tips together (see Pic 6).

Nicolas's building procedure seems to rely more extensively on trying and correcting his actions as these are displayed. To try to account for the difference between the two building procedures, following Zaporozhets (2002a), we may consider perception as a system of actions oriented toward examining the perceived object. These actions include hand actions (as when sensing the contour of an object), eye movements, and language, among others. As a dynamic system, in perception, individuals "grasp," so to speak, the perceived object and, focusing on the whole object or parts of it, single out attributes or properties of it. These attributes or properties are correlated to other attributes, resulting in complex images. In doing so, the individual "transforms certain properties (or sets of properties) of objects into operational units of perception" (Zaporozhets, 2002a, p. 72). In our example, Krista's procedure would be showing three operational units of perception: the initial inverted delta triangle; the delta triangles (Δ) she adds to the emerging form, and the two-Q-tip form required to complete the inverted delta triangles. We may conjecture that Nicolas's recurrent vocalized difficulties are perhaps related to the still ongoing establishment and recognizance of suitable units of perception.

The Teacher

Trying to use Krista's strategy, Nicolas starts building Term 4 by adding triangles. He is counting the triangles in an emerging form having three triangles when the teacher arrives and sits between Krista and Nicolas.

11. Nicolas: (*Talking to the teacher*) I cannot do it.
12. Teacher: Okay, which one can't you do? Tell me where you are at.
13. Nicolas : The fourth [term].
14. Teacher: The fourth? Okay . . .
15. Nicolas: No. That's the third [term].
16. Teacher: Okay, so, we are going to do the fourth [term] together. (*In an encouraging tone*) Okay, reproduce this one (*He points to the model of Term 4 on the sheet; see Figure 5, Pic 1*). Put them [the Q-tips] as they should go. Look, put them as they are (*He takes two Q-tips and forms a "V" in front of Nicolas, while Nicolas follows attentively; see Pic 2*) . . . Okay (*pointing to the "V" form on the desk*). Now, you continue that one [on the model] here (*meaning with the Q-tips on the desk*). I have put two [Q-tips]; I put this one and this one (*pointing twice consecutively to the corresponding lines in the model*). You, you're going to continue; put it [the next Q-tip] on the top (*With the index finger of his right hand, he points to the top of the "V" and, to emphasize the Q-tip's position, he makes a sliding gesture from [his] left to right, while Nicolas follows the gesture with his eyes; see Pic 3*). You'll see that this is going to work.

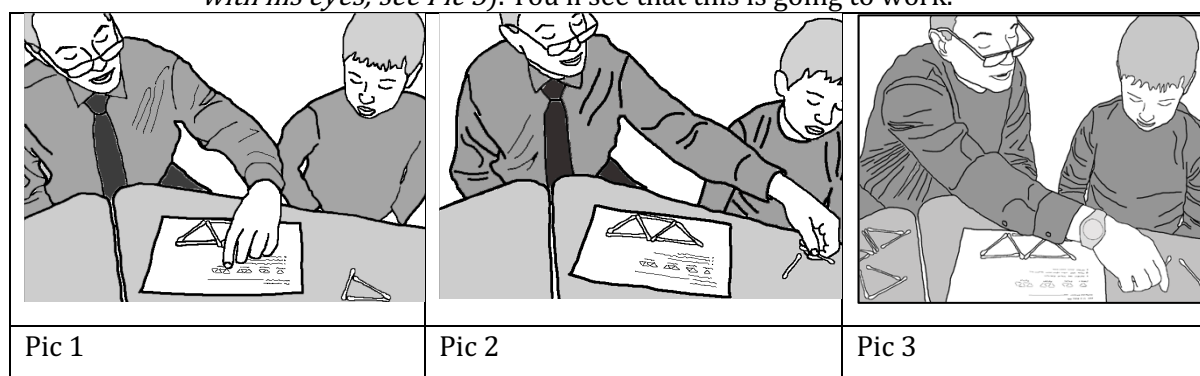


FIGURE 5. The teacher and Nicolas working together on the construction of Term 4

We see that to encourage Nicolas, the teacher starts the construction of Term 4 by referring to Term 4 in the model (Figure 5, Pic 1), then by placing the first two Q-tips of the form (Figure 5, Pic 2). The Q-tips appear to form a "V." But the teacher does much more than that. The teacher makes a *correspondence* between lines in the model and the Q-tips of the in-progress reproduced Term 4. To do so, the teacher uses not only gestures but language too. The teacher's language and gestures allow for a convergence in perception: both the teacher and the child can attend to the same objects on the desk. Although certainly with different meanings and understandings, their attention comes together. Nicolas follows attentively the teacher's embodied actions. Pics 1, 2, and 3 show how he moves his body and his head to follow the teacher and what the teacher points to on the desk. Through language and vivid intonations, the teacher encourages Nicolas to continue and invites him to put the third Q-tip on *top* of the "V" form to close the triangle (although the term is not mentioned yet). In Line 16, he says: "put it [the next Q-tip] on the top" and, to emphasize the position even more, he makes a sliding horizontal gesture with the index finger of his right hand (Pic 3).

At this point Nicolas grabs a bunch of Q-tips and gladly adds the Q-tip on top of the "V" form to make a triangle. The teacher waits for him to finish the action. The teacher continues in an encouraging tone and says:

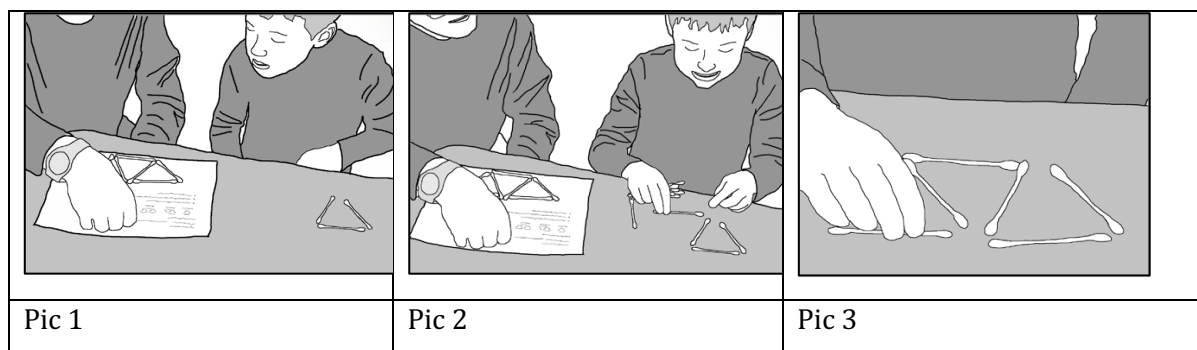
17. Teacher: Okay. Kay. What is next? You are here (*He points to the model (see Figure 6, Pic 1) and Nicolas follows with the eyes. While keeping his hand on the model in a pointing gesture, the teacher continues*). Now you want to add a horizontal (*With a smile on his face, Nicolas adds a Q-tip as shown in Pic 2*). Okay. (*Moving his hand from the model and pointing vaguely to the emerging part of the form the teacher says*) And you want to add the other [Q-tip] there.
18. Nicolas: (*He moves his hand over the form and is about to place a Q-tip beside the upmost right Q-tip of the form, which would result in a double side in the last triangle. He quickly changes his mind and adds it to the right of the form, completing a new triangle*).
19. Teacher: Bravo! You got it! Okay, continue . . .
20. Nicolas: (*He takes a Q-tip, moves it around the form, trying to find a place for it. He tries first to put the Q-tip to the right of the emerging form, on top, in a horizontal manner (see Pic 3); he hesitates, tries to put the Q-tip following an oblique orientation, and hesitates. The teacher intervenes to help him decide.*)
21. Teacher: You have already put this (*pointing to the horizontal line in the second triangle of Term 4 in the model (see Pic 4)*). Then, making a sliding horizontal gesture on top of the emerging form (*see Pic 5*), he continues) Now, in which direction do you want to go?

In Line 21, we see the teacher suggesting through a sliding gesture to add the Q-tip in a horizontal position. However, Nicolas adds the Q-tip in an oblique direction, making a “V.” Then he quickly adds another Q-tip horizontally on top, closing the triangle.

22. Teacher: Okay, there you go! It’s very good. Bravo!

Now, in a confident manner, Nicolas adds the next oblique Q-tip and then a Q-tip in a horizontal position. He smiles at his production. The teacher turns now to Krista to help her with the forms and encourages the children to engage in the construction of Terms 5 and 6.

There was no model shown for these terms; the children were supposed to come up with a strategy to produce them. The teacher left for a moment to answer an intercom call from the school office. In the meantime, Nicolas played a bit with the Q-tips, and added a triangle on top of Term 3. At this moment the teacher comes back to check on the children and Nicolas tells him, “Here is a beautiful ship!” The teacher laughed. Then, the teacher encouraged the children to work together and went to check on another team. After a while, the children came back to the mathematical task. Nicolas quickly constructs Term 5 going from left to right and counting aloud the triangles as he builds the term. He says: “1, 2, 3, 4, 5.” He does the same for Term 6. Pic 6 shows Nicolas’s Terms 5 and 6.



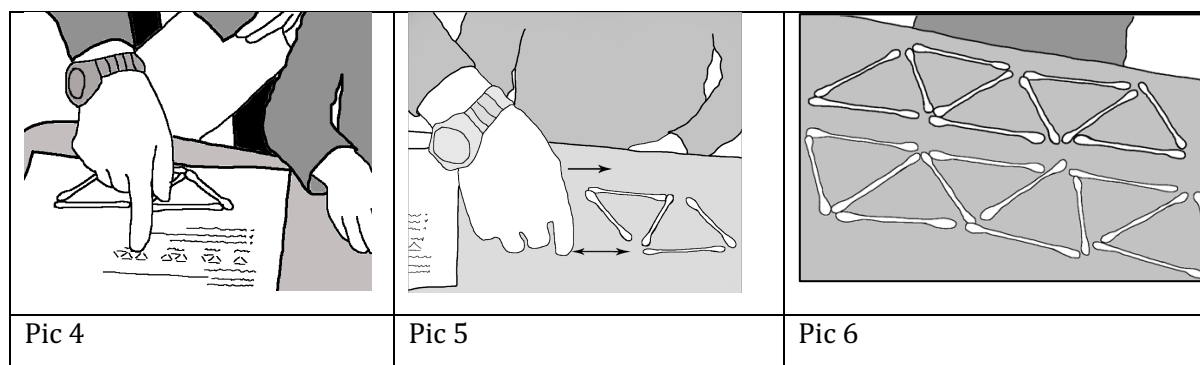


FIGURE 6. The next part of the construction of Term 4 and the final construction of Term 5 and Term 6.

The previous passages can be interpreted as showing the path Nicolas followed in building a schema that allows him to reproduce and expand the terms of the given sequence. Depending on our understanding of embodiment, the building of the schema can be interpreted and understood differently.

From the traditional subjectivist epistemology, Nicolas moved from sense-data and sensorimotor actions to the conceptual realm. Using gestures, tactility, movement, and material objects, he *created* a conceptual schema that allows him now to deal with the subsequent terms of the sequence. The teacher appears here playing an instrumental role, perhaps even interfering too much with the child's progressive conceptualization of the generalization process, which, in this view, should be the responsibility of the student—part and parcel of his own intellectual development and autonomy (Piaget, 1973).

From the dialectical materialist epistemology advocated here, the interpretation goes in a different direction. The understanding of the interaction between the teacher and the students is not based on the idea of student freedom and autonomy but in an ethical engagement that features a commitment towards each other and the unfolding activity they produce. This is a cornerstone of the theory of objectification (Radford, 2008, in press). More precisely, the children and the teacher are conceived of as participating in a joint activity that is at the same time *objective* and *subjective*.

This activity is objective in the sense that it is not a mere contemplative mental activity. It is an activity that is carried out with material *objects*—in this case, the Q-tip forms. In this sense, objective activity means material-object-based activity. But this activity is objective in a second and more important sense. To stop in the first sense would amount to making Feuerbach's mistake mentioned in a section above. It would amount to considering the material objects *as such*, without attending to the cultural conceptual realm that subsumes them (like focusing on objects of consumption without taking into consideration their system of cultural production). This activity is objective in a second sense: like all teaching and learning activities, the activity deals with cultural-historical *objects of knowledge*. In our example, these conceptual objects of knowledge include *cultural-historical manners* in which to perceive the terms of a sequence, how to talk about them, and how to make generalizations. These cultural-historical manners in which to think of sequences in a mathematical way are *objects of knowledge* that were there, in the children's and the teacher's culture. They are part of a curriculum and are recorded in books, pedagogical guides for teachers, etc. But before the commencement of the lesson that morning, these conceptual culturally and historically constituted objects of knowledge were not yet objects of consciousness and thought for our Grade 1 children. In order to appear to the children's consciousness, these objects (the cultural-historical mathematical manner in which to think about sequences) have to be made *sensible*. This is what the activity does. In direct teaching, the teacher, through his/her own deeds, makes these objects of knowledge sensible to the students. But the *subjective* dimension of the students is not taken into consideration. The teacher does all the work; the students are confined to hear, to follow, to obey. The objects of knowledge are not grasped *subjectively* (this is, rephrased, Marx's criticism of Feuerbach's

materialism: the emphasis is on the object (in our case, the *object of knowledge*) to the detriment of the subject). Our lesson goes into a different direction. In our lesson the teacher *engages the students*. He makes room for the students to activate themselves. He tries to open a space for the students to grasp the conceptuality of the objects of knowledge at stake in the lesson. This is what the teacher does when, in Figure 5, Pic 3, he suggests to Nicolas a way to start: he suggests starting with forming a "V." He also suggests looking at the forms in an analytical way. The terms are made of lines, and one can think of them as ordered in a *certain manner* (some are horizontal, as the teacher suggests through gestures and later through words). He also suggests establishing a correspondence between the model and its reproduction; the correspondence proceeds bit by bit, by parts, from left to right, as in Line 21: "You have already put this (*pointing to the horizontal line in the second triangle of Term 4 in the model (see Pic 4 in Figure 6)*)." In doing so, the teacher contributes to disclosing a manner by which to perceive the forms in a certain rational order. As one of my reviewers put it, the teacher offers Nicolas a way of perceiving "the logically defiant Vs, and not the perpetually seductive triangles," which will play a fundamental role when, in a few years, the students will make sense of the coefficient "2" in the symbolic formula " $2n+1$."

Coming back to our excerpts, we see that, in the disclosing of the new perception of the terms, Nicolas makes an important contribution too. To the deeds of the teacher, Nicolas responds positively, and, taking the initiative, starts adding Q-tips to the emerging form (see Figure 6, Pics 2, 3, and 6). In terms of the psychology of perception mentioned above, the joint work of the teacher and the students leads to new "operational units of perception" (Zaporozhets, 2002a, p. 72). The result is that the terms and their parts acquire for the students a sense they did not have before the arrival of the teacher.

To better understand the appearance of the new "operational units of perception," and new organization of actions, let me dwell on the teacher and the students' work. Nicolas and the teacher are engaged in an objective activity where both are fully and symmetrically participating. In previous work *joint labor* is the term that I have used to refer to this symmetrical engagement of teachers and students, where they work hand in hand and where they learn from each other (see Radford, 2016). Embedded in this objective activity a social, sensuous, and material process is taking place: a process of *object-ification*, or (etymologically speaking) a process of *encountering* a cultural-historical *object* (i.e., a target cultural-historical mathematical manner of thinking about sequences). In the course of the process of *object-ification*, the cultural-historical conceptual object is becoming bit by bit an object of consciousness—a sensible, palpable object, that in its contingent sensibility opens itself to the subject and becomes *touchable* so to speak through gestures, language, perception, tactility, body position, and movement. The subject's encountering of the object (i.e., the target manner of thinking mathematically) does not announce the closure of meaning, as something finished and already arrived and accomplished. It rather announces a beginning, a birth: the birth of a capacity to act, the emergence on the horizon of the "I can."

But the activity is not only objective. As mentioned above, the activity is also subjective. It is subjective in the sense that the manner of thinking that acquires materiality in the course of the activity results from the students' and the teacher's *activation*. This activation is manifested in the manner in which the teacher and the students engage in joint labor, their reciprocation of efforts, the emotions, and understandings that they produce as joint labor unfolds, etc. But what the subjective activity of the teacher and the students produces is not something purely whimsical or idiosyncratic. What they sense and feel when they touch something is not mere sense-data. On the contrary—what the subjective activity of the teacher and the students produces is recognized by, and makes sense in, the broader domain of cultural mathematical ways of thinking and inquiring conveyed by the curriculum. To continue with our example above, to see the subjective deeds and their results as purely subjective would amount to

positing them in their existence as objects of consumption independent of production, which is, to rephrase Marx again, the sin of idealism.

It is worth noticing that, in the course of the activity, the students' senses were *transformed*. They changed. The touched transforms the touching. The cultural-historical sensed object transforms the sensing subject. Nicolas's perception, for instance, is not the same at the beginning of the activity as it is after having finished building Terms 5 and 6. By sensing the cultural-historical object, Nicolas's perception now perceives the lines differently; the forms now have a meaning they did not have before. He no longer complains that reproducing the forms is a "hard" task. On the contrary, he finds enjoyment in the task. Tactility and movement have changed too. They are endowed with a conceptuality they did not have before. We see that it is not movement that is the root of thinking (algebraic thinking, in this case). It is objective-subjective activity or *praxis*.

Concluding Remarks

In this chapter I have attempted to look at embodiment from a dialectical materialist perspective. This perspective differs from other materialist perspectives that focus on the world of objects and neglect the *subjective* dimension in the object-subject relationship. This perspective also differs from subjectivist perspectives to embodiment that focus on the body and the senses and neglect the *objective* dimension in the object-subject relationship. The first perspective is epitomized in the work of Ludwig Feuerbach and embraced by empiricism. It reduces the subject-object relation to what individuals touch, perceive, smell, hear, etc., in their contact with a (concrete and conceptual) world that is considered to be already there, for the senses to discover. The second perspective, on the contrary, posits each individual as the creator of her own reality, something that the individual constructs as she activates her vital biological predispositions, the fundamental "I cans," like moving, stretching, etc. This perspective reduces reality and the world to something subjective. Sheets-Johnstone may then say that "we formally create space in the process of moving; we qualitatively create a certain spatial character by the very nature of our movement—a large, open space, or a tight, resistant space, for example" (2011, p. 124).

In contrast to the aforementioned objectivist and subjectivist approaches to embodiment, I have suggested a view that draws from Marx's dialectical materialism and his view of the human (Radford, 2011, 2013, 2014a, 2014b; Radford, Bardini, Sabena, Diallo & Simbagoye, 2005). In this perspective, subject and object come together *in activity*. Both are mutually constructed and reconstructed. None of them are considered as static entities. Both are always in motion, in transformation.

To illustrate my point, I have presented excerpts from a mathematical activity in a Grade 1 class (6-7- year-old students) revolving around the reproduction and generalization of a sequence. I argued that the activity involves both material and conceptual *objects* that have been produced culturally and historically. More specifically, the activity involved a cultural *conceptual object*: a manner of mathematical thinking about sequences whose origins can be traced back to ancient civilizations (Mesopotamia, Greece, Rome) and that is featured in contemporary curricula, often in chapters pertaining to the introduction of algebra. What makes the activity *object-ive* is precisely the *historical* and *cultural* nature of its object. It is *not* any alleged claim concerning the universal truth of the object.

At the beginning of the lesson, this cultural-historical form of thinking eluded the children's consciousness. In order for this manner of thinking to become an object of consciousness, the children and the teacher engaged in a sensuous and material activity in a classroom. A key feature of this activity is its collective nature: the teacher and the students are both symmetrically participating in a fully committed way. To emphasize this commitment, which is

of an ethical nature, I was led to consider the teacher's and students' activity as more than a reciprocation and coordination of actions. It led me to consider the teacher's and students' activity as *joint labor*.

Joint labor included *semiotic components* such as the teacher's and the students' gestures, actions-with-concrete-objects, movement, tactility, perception, body position, and prosody (e.g., intonation and pitch). In the course of joint labor, the general cultural-historical manner of thinking about sequences acquired bit by bit concrete, cultural, and subjective determinations. In the example discussed here, these determinations included perceiving the given terms as made up of a triangle followed by "V" forms (of course, other cultural and subjective determinations are possible as there are different ways in which to perceive the terms of a given sequence). Hence, through the semiotic and sensible teacher-student joint labor emerged new general organizations of actions that allowed Nicolas and Krista to reproduce the given terms of the sequence and to generalize those actions to other non-visible terms.

Certainly, both the perception of the terms and the procedure to reproduce and expand the terms still need to acquire a more pronounced salience in the students' consciousness. They need to be sharpened through further joint labor and new articulations and expansions of its semiotic components (e.g., a more refined and precise use of language). For the manner, depth, and intensity in which an object appears as an object of consciousness are consubstantial with the complexity of joint labor, which is what makes possible for such an object to become an object of consciousness and thought (Radford, 2018). At this point in the ontogenetic development of the students, the sharpening of the object of consciousness might need further articulation through the use of language (e.g., as in *naming* the central operational units of perception).

At any rate, this "procedure" of reconstruction and generalization of the terms of the sequence comes close to the psychological concept of "image" (Zaporozhets, 2002a) and to Piaget's concept of the "schema" (Beth & Piaget, 1966), both relying on the idea of internalized actions. I prefer to think of this "procedure" as an always emergent, fluid, contextual, and sensuous articulated entity consisting of perceptions and kinesthetic living actions of the sensing individual as he/she engages in practical activity with objects. This "procedure" comes to life in the very moment in which it is accomplished. Its locus is not *in* the body nor *in* the objects, but *in between*; that is to say, in the *activity* that brings the sensing subject and the sensed objects together. We can call this dynamic embodying entity a *simile*. It is always different, yet similar. It is the conciliation of its irreconcilable opposites: the new and the old; the past and the future. The simile is bearer of the aesthetic and emotional unity of objects and bodies in practical activity.

Let me summarize the dialectical materialist conception of embodiment that I have sketched here. Embodiment, I want to suggest, is the expression and content of

- (1) the subjective dimension of objective activity (*gegenständliche Tätigkeit*), and
- (2) the objective dimension of subjective activity.

Embodiment, in short, is the name of the activity-based dialectical relationship between sensing subjects and the cultural-historical sensed objects.

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